Hiermit bestätige ich, dass ich zur Klausur angemeldet und zugelassen bin.
Eine falsche Angabe wird als Täuschungsversuch gewertet.

Unterschrift: ________________
1 “Simple algorithms”

Define a function \texttt{sum} to sum up a list of ints. Please use pattern matching on lists. Here is a demo:

\begin{verbatim}
> sum [1,2,3]
6
\end{verbatim}

\begin{center}
\textbf{Reference solution}
\end{center}

\begin{verbatim}
-- Import not required
import Prelude hiding (sum)

-- Signature not required
sum :: [Int] -> Int
sum [] = 0
sum (x:xs) = x + sum xs
\end{verbatim}
2 “Simple data models”

Declare a data type for ‘boxes’ as follows. A ‘box’ contains items and it has a certain size (i.e., a float for the side length). An ‘item’ can be either another (presumably smaller sized) ‘box’ or ‘filler material’ measured in terms of a float for its weight. (Thus, boxes may be arbitrarily nested.)

Reference solution

```haskell
-- Solutions may also define less types than shown below.
data Box = Box Size [Item]
data Item = BoxItem Box | FillerItem Filler
data Filler = Filler Weight
type Size = Float
type Weight = Float
```
3  “Local scope”

Consider the following code:

\[
\text{test } x \ y \ z = \text{smaller } x \ y \ \&\& \text{smaller } x \ z \\
\text{smaller } x \ y = x < y
\]

Transform the code such that local scope is used for the definition of \textit{smaller}, i.e., \textit{smaller} becomes a local function of \textit{test}. The local definition should only have a single argument. Hint: note that \textit{smaller} is invoked both times with the same first argument.

\begin{verbatim}
Reference solution

\text{test } x \ y \ z = \text{smaller } y \ \&\& \text{smaller } z \\
where
\text{smaller } q = x < q -- y \text{ could be used instead of } q
\end{verbatim}
4 “Parametric polymorphism”

Define a polymorphic function \textit{shiftLeft} including its function signature for shifting all elements of a list to the left with the original head becoming the last element of the resulting list. Here is an illustration:

\begin{verbatim}
> shiftLeft []
[]
> shiftLeft [1]
[1]
> shiftLeft [1,2]
[2,1]
> shiftLeft [1,2,3]
[2,3,1]
\end{verbatim}

Reference solution

\begin{verbatim}
shiftLeft :: [a] -> [a]
shiftLeft [] = []
shiftLeft (x:xs) = xs ++ [x]
\end{verbatim}
“Higher-order functions”

Define the polymorphic function `maybe` which dispatches on a `Maybe` value as demonstrated here:

```haskell
> maybe 0 (1+) Nothing
0
> maybe 0 (1+) (Just 41)
42
```

That is, ‘`maybe b f v`’ evaluates to `b` if `v` is `Nothing` and it evaluates to ‘`f a`’ if `v` is ‘`Just a`’. (This is the standard `maybe` function.)

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**Reference solution**

```haskell
-- Import not required
import Prelude hiding (maybe)

-- Signature not required
maybe :: b -> (a -> b) -> Maybe a -> b
maybe b _ Nothing = b
maybe _ f (Just a) = f a
```
6 “Monoids”

Consider the following code:

```
instance Monoid [a] where
    mempty = []
    mappend = ++
```

What does the shown monoid instance describe? Please, be concise: 140 characters or less.

**Reference solution**

The instance makes list types monoids with the empty list as neutral element and list append as the associative operation.
7 “Functors”

Consider the following data-type declaration for some sort of trees with one constructor for empty trees and another constructor for forking trees with an associated list of values of the parameter type of the datatype constructor:

\[
\text{data} \quad \text{Tree} \ a \ = \ \text{Empty} \mid \text{Fork} \ [a] \ (\text{Tree} \ a) \ (\text{Tree} \ a)
\]

Describe an instance of the type class \textit{Functor} with its member function \textit{fmap}, as needed for the trees at hand.

\begin{center}
\begin{verbatim}
Reference solution

instance Functor Tree
  where
    fmap _ Empty = Empty
    fmap f (Fork as x y) = Fork (map f as) (fmap f x) (fmap f y)
\end{verbatim}
\end{center}
8  “Reasoning”

Here is an attempt at formulating a property for testing `drop`. (Remember, `drop` is the function which drops (‘removes’) the given number of elements of a list.)

```haskell
import Test.QuickCheck

prop_drop x l = length (drop x l) > length (drop (x+1) l)
```

This ‘property’ is not universally true. Give an application of the ‘property’ for which it returns `False`.

Reference solution

```
> prop_drop 1 [42]
False
```

Just for the record, QuickCheck also shows that the property is not satisfied:

```
> quickCheck prop_drop
*** Failed! Falsifiable (after 1 test and 2 shrinks):
0
[]
```
9 “Lazy evaluation”

Consider the following definition of the factorial function:

```haskell
factorial x = product [1..x]
```

Now, also consider the following definition of all non-zero natural numbers:

```haskell
nats = nats' 1
    where
    nats' x = x : nats' (x+1)
```

(Clearly, \( nats \) denotes an infinite list.) Re-define the factorial function to use \( nats \) rather than the “..” notation. Hint: you may also need the \( Prelude \) function \( take \) for taking (‘selecting’) a given number of elements of a list.

Reference solution

```haskell
factorial x = product (take x nats)
```
10 “Monads”

Consider the following definition of \textit{return} of a \textit{State} monad.

\begin{verbatim}
-- Data type for the state monad
newtype State s a = State { runState :: s \rightarrow (a,s) }

-- Monad instance for State
instance Monad (State s) where
  return x = State (\s \rightarrow (x, s))
  c >>= f = ... -- omitted for brevity
\end{verbatim}

What does the definition of \textit{return} model? \textit{Please, be concise: 140 characters or less.}

\begin{quote}
\textbf{Reference solution}

A value becomes a state-aware computation by passing on unmodified the incoming state.
\end{quote}